formance sensitivity occurs for variations in aircraft weight and specific fuel consumption. However, fuel consumption appears to be relatively insensitive to variations in thrust and atmospheric variations.

Path Adjustment

Performance sensitivity may be compensated by adjusting the nominal trajectory to be fuel optimal for the specific variation under consideration. As a first step, optimal cruise points are determined for each variation using Eq. (3). Only variations in the drag coefficients, aircraft weight, and atmospheric conditions cause a shift in the cruise point. The cruise point shifts upward as the drag coefficients and aircraft weight increase, and it shifts downward as the parameters decrease; while a Hot Day atmosphere causes the cruise point to shift upward and a Cold Day atmosphere causes a downward shift. Using the values for $(\sigma_c D_c / V_c)$ calculated at these adjusted cruise points, the adjusted minimum fuel trajectory for each variation is determined. The fuel performance sensitivity for each variation is given in Table 3. Path adjustment causes very little, if any significant improvement in fuel consumption for any of the parametric or atmospheric variations. For example, when aircraft weight is increased by 20% along the nominal path, fuel consumption increases by 23%. However, when the nominal trajectory is adjusted to be fuel optimal for a 20% weight increase, the fuel consumption is increased by 22% representing a 1% improvement. Obviously, adjusting the nominal trajectory to be fuel optimal for the parameters examined here fails to provide significant improvement in fuel consumption.

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Real Model Following Control

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Introduction

THE technique of real model following control (RMF) has been shown to be amenable to the solution of many aircraft control problems. Commencing with the work of Kalman[1] and Tyler[2] extensive use has been made of linear optimal control theory in the design of RMF controllers. Available designs include the partial state feedback controller of Winsor and Roy[3] and the stability augmentation and mode decoupling controller of Yore.[4] While optimal control theory provides an extremely flexible synthesis technique a structural approach may, nevertheless, provide a superior design. In particular, it may be possible to achieve 'perfect' following, i.e. perfect matching of the dynamics of the compensated plant to those of the model, without rescourse to a high-gain controller. Further, an algebraic control law may be readily combined with a parameter estimation scheme to provide an adaptive capability.

Following the work of Erzberger[5] on the implicit model following problem various structural approaches have been adopted in synthesizing RMF controllers. Curran[6] exploited the concept of 'equicontrollability' to present an approximate design technique. An asymptotic RMF control law was derived by Chan[7] for the class of plants and models whose output vectors are identically their state vectors. Landau and Courtiol[8] have considered the adaptive model following problem for the same class of systems. Lowe,[9] using the second method of Lyapunov, derived an asymptotic control law applicable to single variable systems. In this paper a control law is obtained for a class of linear multivariable systems and the necessary and sufficient conditions for perfect following are derived. This control law reduces to that of Lowe for single variable systems and the derivation parallels the Lyapunov synthesis technique.

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Asymptotic Solution

Consider the linear time invariant multivariable plant

$$\dot{x}_p = A_p x_p + B_p u_p$$

$$y_p = C_p x_p \tag{1}$$

where x_p is an n state vector, u_p an m input vector and y_p an m output vector. A_p, B_p and C_p are matrices of appropriate dimensions. It is assumed that the triple (A_p, B_p, C_p) is controllable and observable. The prespecified linear time invariant model is described by

$$\dot{x}_m = A_m x_m + B_m u_m$$

$$y_m = C_m x_m \tag{2}$$

where the dimension of x_m, the state vector, is arbitrary, u_m is an m input vector and y_m is an m output vector.

The model following error e is defined as

$$e(t) = y_m - y_p \tag{3}$$

The design problem is to synthesize u_p such that $e(t) \to 0$ in the steady state. For a given plant and model the necessary and sufficient conditions for the existence of a perfect RMF control law must also be derived.

The time derivative of the error is given by

$$\dot{e} = \dot{y}_m - \dot{y}_p \tag{4a}$$

$$= C_m A_m x_m + C_m B_m u_m - C_p A_p x_p - C_p B_p u_p \tag{4b}$$

Let the input to the plant be decomposed as

$$u_p = K e + u_p^* \tag{5}$$

where K is a gain matrix to be determined.

Substituting Eq. (5) into Eq. (4b) yields

$$\dot{e} = -C_p B_p K e + C_m A_m x_m + C_m B_m u_m - C_p A_p x_p - C_p B_p u_p^* \tag{6}$$

It is required that

$$\lim_{t \to \infty} e(t) = 0 \tag{7}$$

Let K be chosen such that all the eigenvalues of $(-C_pB_pK)$ have negative real parts. Arbitrary eigenvalue assignment is possible provided det $(C_pB_p)\neq 0$. If det $(C_pB_p)=0$ some of the eigenvalues may be placed at the origin at the expense of a steady-state error. Equation (7) may thus be satisfied if det $(C_pB_p)\neq 0$ and

$$C_mA_m\boldsymbol{x}_m + C_mB_m\boldsymbol{u}_m - C_pA_p\boldsymbol{x}_p - C_pB_p\boldsymbol{u}_p^* = 0 \qquad (8)$$

which yields a unique solution for $\boldsymbol{u}_p^*$. If C_pB_p is singular, but the steady-state error due to the zero eigenvalues of $(-C_pB_pK)$ is acceptable, the best approximate solution is the least squares sense for $\boldsymbol{u}_p^*$ is given by

$$\boldsymbol{u}_p^* = (C_pB_p)^+[C_mA_m\boldsymbol{x}_m + C_mB_m\boldsymbol{u}_m - C_pA_p\boldsymbol{x}_p] \qquad (9)$$

where + denotes the pseudoinverse of a matrix and it is assumed, for the present, that $C_pB_p\neq 0$. Substituting Eq. (9) back into Eq. (8) yields the necessary and sufficient conditions for RMF as

$$\Gamma C_mB_m = \Gamma C_mA_m = \Gamma C_pB_p = 0 \qquad (10)$$

where $\Gamma=[I-(C_pB_p)(C_pB_p)^+]$. The RMF control thus takes the form

$$\boldsymbol{u}_p = K_p\boldsymbol{x}_p + K_m\boldsymbol{x}_m + K_u\boldsymbol{u}_m \qquad (11)$$

where

$$K_p = -KC_p - (C_pB_p)^+C_pA_p \qquad (12a)$$

$$K_m = KC_m + (C_pB_p)^+C_mA_m \qquad (12b)$$

$$K_u = (C_pB_p)^+C_mB_m \qquad (12c)$$

The model following configuration is depicted in Fig. 1. When $C_pB_p=0$ a similar control law can be derived by decomposing $\boldsymbol{u}_p$ as

$$\boldsymbol{u}_p = K_1e + K_2\dot{e} + \boldsymbol{u}_p^* \qquad (13)$$

and obtaining K_1,K_2 and u_p^* such that the limit of $e(t)=0$. A nontrivial solution will exist for u_p^* provided $C_pA_pB_p\neq 0$. If $C_mB_m\neq 0$ $\boldsymbol{u}_p$ will contain a term involving $\dot{\boldsymbol{u}}_m$ which may not be acceptable. If $C_pB_p=0$ and $C_pA_pB_p=0$ a control law can be synthesized by including higher derivative terms of $e(t)$ in the decomposition of $\boldsymbol{u}_p$. On implementation of the control law of Eq. (11) the closed loop poles of the plant are given by the zeros of det $[sI-A_c]$ where

$$A_c = A_p - B_pKC_p - B_p(C_pB_p)^+C_pA_p \qquad (14)$$

and will not necessarily all lie in the left half of the complex plane. The stability of the controller must therefore be checked independently. If C_pB_p is nonsingular it can be verified that m eigenvalues of the closed loop system are given by the eigenvalues of $(-C_pB_pK)$. This can be shown by a linear transformation of the closed loop state equations by

$$Q=\begin{bmatrix} C_p \\ \hline D \end{bmatrix}$$

where D is arbitrarily chosen such that Q is nonsingular.

Simulation Results

The application of the RMF control law discussed in the previous section is illustrated by a simulated aircraft example. The problem considered is that of achieving lateral-directional decoupling of a T-33. Hall[10] first treated this problem by a 'response-feedback' approach and it was subsequently treated by Cliff and Lutze[11] using geometric decoupling theory. The plant is described by the matrices

$$A_p=\begin{bmatrix} -3.18 & 0.0 & 0.63 & -10.6 \\ 1.0 & 0.0 & 0.0 & 0.0 \\ -0.06 & 0.0 & -0.27 & 4.18 \\ 0.022 & 0.644 & -0.998 & -0.151 \end{bmatrix}$$

$$B_p=\begin{bmatrix} -14.4 & 1.5 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & -2.59 & -0.96 \\ 0.0 & 0.037 & 0.0 \end{bmatrix}$$

$$C_p=\begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

with $\boldsymbol{x}_p=(p_p,\phi_p,\beta_p)^T$ and $\boldsymbol{u}_p=(\delta a_p,\delta r_p,\delta p_p)^T$.

The matrices for the noninteractive model are given by

$$A_m=\begin{bmatrix} -3.18 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.27 & 0.0 \\ 0.0 & 0.0 & 0.0 & -0.151 \end{bmatrix}$$

$$B_m=\begin{bmatrix} -14.4 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.96 \\ 0.0 & 0.037 & 0.0 \end{bmatrix} \qquad C_m=C_p$$

with $\boldsymbol{x}_m=(p_m,\phi_m,r_m,\beta_m)^T$ and $\boldsymbol{u}_m=(\delta a_m,\delta_m,\delta p_m)^T$.

It is easily verified that perfect model following can be achieved. Assigning eigenvalue $-5.0,-5.0,-2.5$ to $(-C_pB_pK)$ yields a suitable K as

$$K=\begin{bmatrix} -0.347 & 0.0 & 7.038 \\ 0.0 & 0.0 & 67.568 \\ 0.0 & -5.208 & -72.125 \end{bmatrix}$$

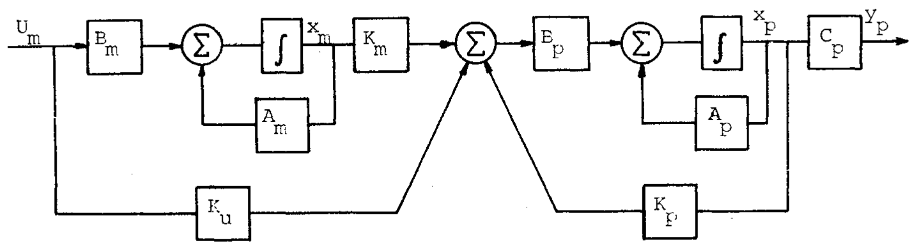

Fig. 1 Real model following configuration.

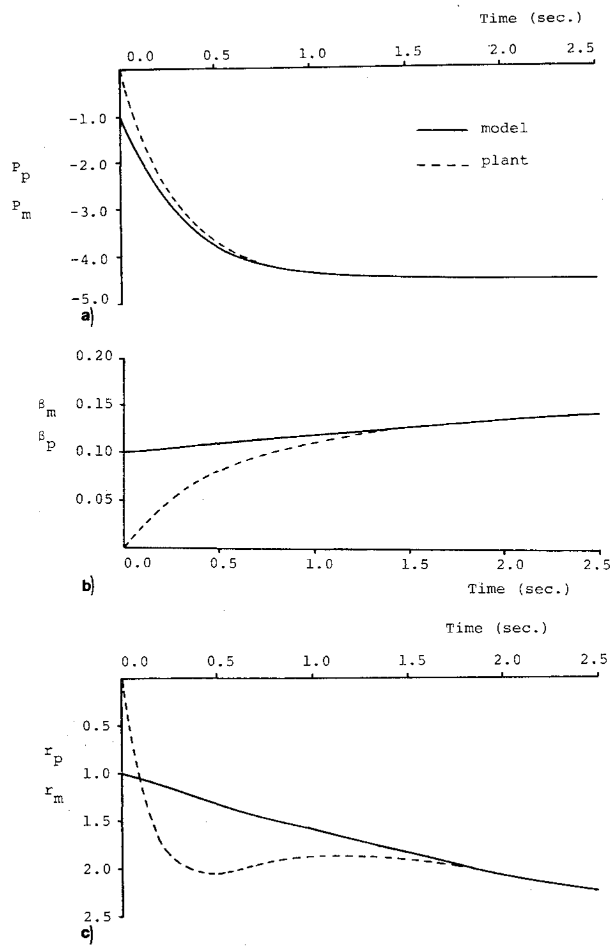

Fig. 2 RMF simulation results: a) $\delta a_m = u(t)$; $\delta r_m = \delta p_m = 0$; b) $\delta r_m = u(t)$; $\delta a_m = \delta p_m = 0$; and c) $\delta p_m = u(t)$; $\delta a_m = \delta r_m = 0$.

Hence the R.M.F. gains are computed from Eq. (12) as

$$K_p = \begin{bmatrix} 0.065 & -0.181 & 2.853 & -7.349 \\ -0.595 & -1.741 & 26.973 & -63.487 \\ 1.542 & 4.696 & -67.844 & 71.469 \end{bmatrix}$$

$$K_m = \begin{bmatrix} -0.126 & 0.0 & 0.0 & 6.613 \\ 0.0 & 0.0 & 0.0 & 63.487 \\ 0.0 & 0.0 & -4.927 & -67.115 \end{bmatrix}$$

$$K_u = \begin{bmatrix} 1.0 & 0.104 & 0.0 \\ 0.0 & 1.00 & 0.0 \\ 0.0 & -2.698 & 1.0 \end{bmatrix}$$

The control law results in the closed loop plant having the characteristic polynomial $s(s+5.0)(s+5.0)(s+2.5)$.

To verify that perfect model following is achieved and that the plant is decoupled in the steady state the controlled system was simulated using the IBM System/360 C.S.M.P. The initial conditions on the plant and the model were $x_p(0)=0$ and $x_m(0)=(-1.0, 0.0, -1.0, 0.1)^T$ respectively. Asymptotic model following and decoupling were indeed achieved. The responses of the decoupled modes of the plant and the model to step inputs to the model are shown in Fig. 2.

Conclusions

An RMF control law which does not require that the plant model have the same order has been developed in this paper. Though this control law is only a partial solution to the problem it has been found to be applicable to a significant number of practical systems and its application has been illustrated by an aircraft lateral control problem. Work on developing an RMF control law which permits arbitrary placement of the closed loop poles of the plant is presently being undertaken.

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Method for Developing "Around-the-Clock" Gust Spectra

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Nomenclature

G_{ue}	= number of gusts per mile (km) encountered exceeding a given value
P	= proportion of flight distance flown in turbulence
$G_{(0)}$	= frequency of occurrence of gusts per flight mile (km) above zero gust velocity
u_e	= gust velocity for which exceedances are to be calculated, actually the vertical component of that gust velocity, fps (mps)
b	= slope of the curve on a semi-log plot for the basic distribution of turbulence
k	= ratio of the intensity of turbulence at any altitude to the basic intensity
$G_{1(ue)}$	$= P_1 G_{1(0)} e^{-ue/b_1 k_1}$
$G_{2(ue)}$	$= P_2 G_{2(0)} e^{-u_e/b_2 k_2}$

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